

187 §3- Discrete Mathematics - Final Exam

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Name _____

Please show all your work, not just final answers, and *explain what you are doing*. Answer on your blue book. Indicate clearly in each page the question you are working on.

Good luck, and happy holidays!

1. On a poll of 50 people, none of whom likes fish, it was found that 33 like chicken, 20 like pork, and 18 like beef. None of them like all three kinds of meat, eight of them like both chicken and pork, nine like pork and beef, and seven like chicken and beef. Find how many of them are vegetarians.
2. Use resolution to prove that the following propositional statement is a tautology:

$$(p \Rightarrow q) \Rightarrow ((q \Rightarrow r) \Rightarrow (p \Rightarrow (p \wedge q \wedge r))).$$

[If needed, feel free to use each clause several times. Remember that resolution only allows us to prove that a sentence is *false*.]

3. True or false? (For each, circle “T” or “F”. No reason required.)
 - (a) T F A proof of “ $A \Rightarrow C$ ” proves “ A and $B \Rightarrow C$ ”.
 - (b) T F A proof of “ $(A \text{ or } B) \Rightarrow C$ ” proves “ $A \Rightarrow C$ ”.
 - (c) T F $x > -3 \Rightarrow x \geq -3$.
 - (d) T F $|b| > 2 \Rightarrow b \geq 2$.

4. Resolve the following conjecture:

If x is rational and y is irrational, then xy is irrational.

5. Prove that for any positive integer n , we have that $6 \mid (7^n - 1)$.

6. Prove by induction that for any positive integer n ,

$$\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}.$$

7. Each of your five favorite authors has written exactly 6 books. A friend wants to give you as a gift a package consisting of 5 of these books.
- (a) In how many ways can this be done, if repeating books is possible?
 - (b) In how many ways can this be done, if you are to receive 5 different books?
 - (c) In how many ways can this be done, if you are to receive a book from each author?
8. Let c be the GCD of 119 and 901.
- (a) Find c .
 - (b) Find integers x and y such that $119x + 901y = c$.
9. Prove that for all $n \geq 8$, the equation

$$3x + 5y = n$$

has a solution with x, y nonnegative integers.

10. Suppose that A is a set and $f : A \rightarrow A$ is a given function, with the property that $f(f(x)) = x$ for all $x \in A$. Prove that f is a bijection.

The following are extra credit problems. Only work on them once you are done with the previous questions.

- i. Find the number of nonnegative integer solutions of

$$x + 2y + 3z = 54.$$

- ii. Write a positive integer. You will receive a small extra credit bonus if the number you wrote is the smallest number that *nobody else in the class* wrote.