Name: _____ Student ID number: _____

Ma 187 §3-DISCRETE MATHEMATICS – Midterm Exam 1
Instructor: Andrés E. Caicedo Sept. 27, 2011

Good luck!

1. These might have grammatical mistakes, given our conventions. If they do not, say "correct." If they do, note what is wrong with them.

(a) $5 \subset [0,7]$ (b) $x \in S \implies x \in T$ (c) $\{0,5\} \cap (1,\infty)$ (d) $3x < 6 \implies \{x \mid x < 2\}$

2. Here is a form: $P \implies Q$.

(a&b) Give two other forms that are logically equivalent to it.
(c) Give its converse.
(d) Give its negation.

3.

(a) Define tautology.
(b) State one.

4. Are these pairs of sentences equivalent? If so, just say "Yes". If not, say "No" and give a **specific** counterexample. [Assign particular values to each letter.]

(a) $x^2 > 4x$, $x > 4$.
(b) $t < 2$, $z < 2$.
(c) not($y < -3$ or $y > 8$), $y \geq -3$ and $y \leq 8$.

5. Draw a Venn diagram for $(A \cap B \cap C)^c$ and another one for $A^c \cup B^c \cup C^c$. What do you conclude?

6. Use resolution to show that the following set of clauses is unsatisfiable:

$p \Rightarrow q$ $q \Rightarrow r$ $\neg(p \wedge q \wedge r)$ p

7. True or false? (For each, circle "T" or "F". No reason required.)

(a) T F A proof of "$A \implies C$" proves "A and $B \implies C$".
(b) T F A proof of "(A or B) $\implies C$" proves "$A \implies C$".
(c) T F $x > 5 \implies x \geq 5$.
(d) T F $|b| > 5 \implies b \geq 5$.

8. Create a truth table, with all appropriate columns (Do not skip columns!) for determining if "(not A) $\implies B$" is logically equivalent to "A or not B". At the end, say if they are or are not logically equivalent, and why.

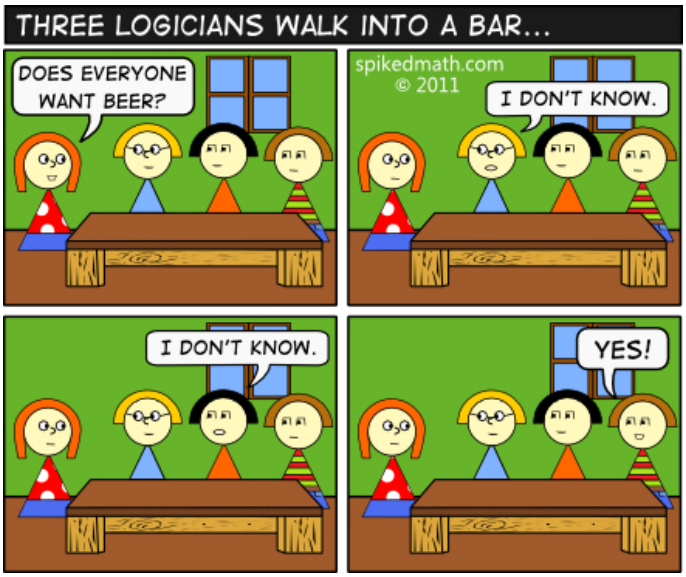
9.

(a) Give examples of sets R and S such that $R \subset S$.
(b) Give examples of sets R and S such that $R \subset S^c$.
(c) The statement "$R \subset S$ or $R \subset S^c$" is false. Give a counterexample.

10.

(a) Write down the definition of $5n + 1 = O(n)$.
(b) Prove that $5n + 1 = O(n)$ is in fact true.

Extra credit problem. Only work on this if you are done with the rest. Explain the following "joke."



(From http://spikedmath.com/445.html)