Name: ______________________________

Ma 187 §3-DISCRETE MATHEMATICS – Midterm Exam 2
Instructor: Andrés E. Caicedo Nov. 2, 2011

Please answer in your blue book.

Good luck!

1. Show that $0.134134134134134\ldots$ is a rational number.

2. Resolve the following conjecture:

If x is irrational and positive, then $\sqrt{x}$ is also irrational.

3. Consider the following statement:

If x is irrational and y is irrational, then xy is irrational.

(1) State the negation of the statement above.
[Be careful, note that some quantifiers may not be explicitly written down but need to be mentioned explicitly in the negation.]
(2) Prove the negation.

4. Resolve the following conjecture:

If $a|b$ and $b|c$ then $a|c$.

5. Prove by induction that $3|(4^n - 1)$.

6. Prove:

If n is an integer, and n^2 is a multiple of 3, then n is a multiple of 3.

[Make sure only results covered in lecture are used as prior. This problem can be done in several ways, using techniques we have not yet seen. These arguments are not acceptable here.]

7.

(1) Use the Euclidean algorithm to find the Greatest Common Divisor of 187 and 253.
(2) Find integers x, y such that

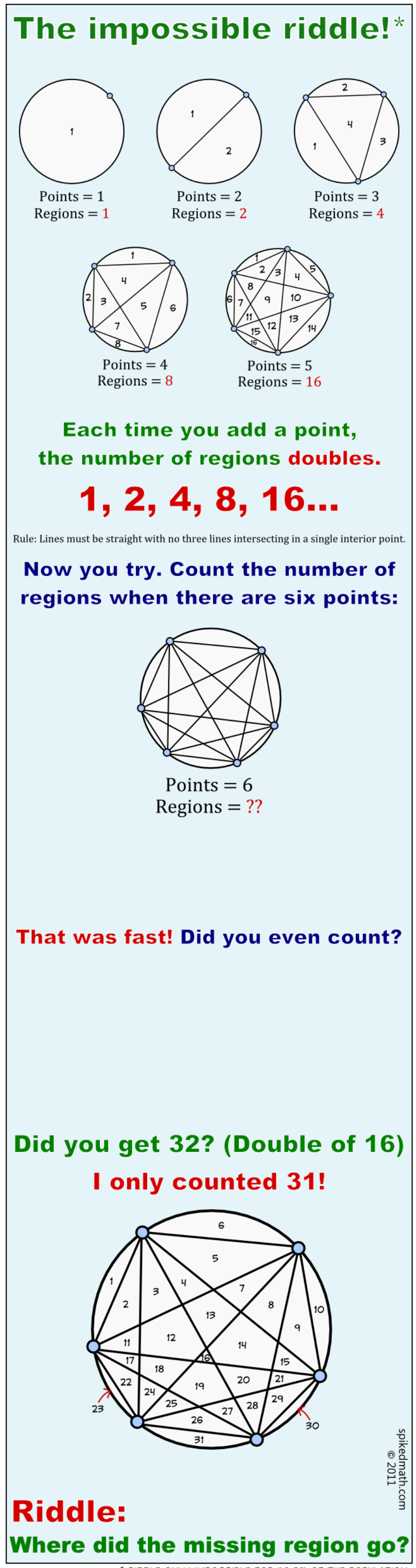
$$187x + 253y = \mathrm{GCD}(187, 253).$$

8. Analyze the following argument; in particular, if the argument is incorrect, indicate explicitly the line where the argument goes wrong, and explain how the proof could be fixed. Even if the overall argument is correct, some lines may require additional explanations, or small corrections. If the statement itself is false, but a small fix may make it true, indicate how to change the statement, and/or how to modify the given proof.

$$1 + 3 + 5 + \cdots + (2n - 1) = n^2.$$

(1) We argue by induction on n.
(2) The base case is clear.
(3) Assume the nth case of the identity, and proceed to prove the $(n + 1)$st case:
(4) If $1 + 3 + \cdots + (2n + 1) = (n + 1)^2$, then
(5) $(1 + 3 + \cdots + (2n - 1)) + (2n + 1) = (n + 1)^2$, then
(6) $n^2 + (2n + 1) = (n + 1)^2$, by the inductive assumption,
(7) and the last line is clearly true.

Extra credit problem. Only work on this if you are done with the rest. Explain the following.



(From http://spikedmath.com/449.html)